Multidimensional Time-Domain Macromodels for Microwave Applications

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Abstract—**During the design of microwave circuits there is a significant need for efficient but accurate macromodels of components. These macromodels are derived from physical models of the components and may model either electrical or thermal behavior. However, increased levels of integration and higher power levels in microwave devices and modules have produced a need to include effects previously neglected during simulations. Accurate prediction of these effects involve solution of large systems of equations, the direct simulation of which is prohibitively CPU expensive.**

In this paper, an algorithm is proposed to form passive reduced-order macromodels of large linear networks that match the characteristics of the original network in time as well as other design parameters of the circuit. A novel feature of the algorithm is the ability to incorporate a set of design parameters within the reduced model.

I. Introduction

Recent advances in fabrication technology have reduced the feature sizes and increased the density of chips. At high signal frequencies, transmission line effects such as delay, coupling and crosstalk, previously neglected during circuit simulation have become prominent. In addition, increased levels of integration are giving rise to thermal design problems due to self-heating and thermal coupling which leads to reliability concerns. These effects, if not predicted at early design stages can severely effect system performance, potentially delaying the design cycle. Accurate prediction of these physical (either electrical or thermal) effects generally requires the solution of large systems of equations, the simulation of which is prohibitively CPU expensive [1]–[4]. Forming a reduced linear model is the key to fast simulation of such large systems of equations.

In addition to reducing the CPU expense of regular simulations, it is also important to predict the response of the designed circuit due to environmental effects, thermal effects, manufacturing variations and fluctuations in the critical dimensions of interconnects such as width and height. These effects induce a change in the overall system equations. It is often not feasible to perform simulation of large circuits due to variations in these parameters. A reduced-order macromodeling technique that can perform analysis on large systems of equations with respect to time and other design parameters of the circuit can aid in addressing these issues.

In the literature, several algorithms were proposed to efficiently simulate large systems of equations by model-order reduction [5]–[7]. It is to be noted that all these techniques perform model reduction with respect to a single parameter (frequency). In [8], [9] methods have been proposed to perform multidimensional *analysis* on large systems of equations. It has to be emphasized that these techniques are simulation techniques and thus do not aid in forming time-domain macromodels. Attempts have been made to form multidimensional time-domain macromodels of linear networks in [10]. However, these techniques do not guarantee passivity of the macromodel. Passivity implies that a network cannot generate more energy than it absorbs, and no passive termination of the network will make the system unstable. Passivity is an important property, because stable but not passive macromodels can lead to unstable or non-physical systems when connected to other passive systems.

In this paper, a technique is proposed that can form reduced-order macromodels of large linear systems w.r.t time and other design parameters of the network while guaranteeing the passivity of the reduced-order macromodel. The algorithm is based on Krylov subspace techniques extended to multiple dimensions. The theoretical approach to form passive multidimensional time-domain macromodels is presented in Sec. II. This is followed by Sec. III, which presents the results and finally the conclusions are presented in Sec. IV.

II. Theoretical Approach

A. Formulation of Network Equations

A general linear passive subnetwork, function of parameters $\lambda_1, \lambda_2 \dots \lambda_n$ can be expressed as

$$\mathbf{C}(\lambda_1 \dots \lambda_n)\frac{dx}{dt} + \mathbf{G}(\lambda_1 \dots \lambda_n)x = \mathbf{B}u_p(t) \qquad (1a)$$

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$$i_p(t) = \mathbf{B}^T x \tag{1b}$$

where, $x(t, \lambda_1 \ldots \lambda_n) \in \Re^N$ is the vector of unknowns in the system, $\mathbf{C}(\lambda_1 \ldots \lambda_n) \in \Re^{N\times N}$, $\mathbf{G}(\lambda_1 \ldots \lambda_n) \in \Re^{N\times N}$ are matrices describing the lumped memory and memoryless elements of the network dependent on $\lambda_1 \ldots \lambda_n$. i_p and u_p denote the port currents and port voltages respectively, p being the number of ports, $\mathbf{B}$ is a selector matrix that maps the port voltages into the node space of the network, N is the total number of variables in the MNA formulation and n is the number of parameters in the network. These variables as described are for an electrical network, however, for a thermal network, x would map to a vector of nodal temperatures and u_p and i_p to port temperatures and heat flows respectively.

B. Computation of the Multidimensional Subspace

The first step of the algorithm is to compute the multidimensional subspace of the system in (1). For that purpose, the block moments of x w.r.t frequency (denoted by $\mathbf{M}_s$) are computed using the procedure described in [7], [11] using Krylov subspace techniques. Block moments of x w.r.t $\lambda_1 \ldots \lambda_n$ (denoted by matrices $\mathbf{M}_{\lambda_i}, 1 \leq i \leq n$) are computed using the technique elaborated in [8]. In the special case where the variation of the matrices $\mathbf{C}$ and $\mathbf{G}$ w.r.t the parameter λ_i is linear, techniques such as Arnoldi [11] can be used to compute the block moments of x w.r.t λ_i to extract better efficiency. Once all the required block moments are evaluated, the multidimensional subspace (denoted by $\mathbf{Q}$) is computed such that

$$\text{colsp}(\mathbf{Q}) = \text{colsp}\begin{pmatrix}\mathbf{M}_s & \mathbf{M}_{\lambda_1} & \ldots & \mathbf{M}_{\lambda_n}\end{pmatrix} \tag{2}$$

This can be achieved by using a standard QR decomposition [11] on $[\,\mathbf{M}_s\ \mathbf{M}_{\lambda_1} \ldots \mathbf{M}_{\lambda_n}\,]$.

In the case when cross-derivatives are required to preserve the network response, the multidimensional subspace is modified to include the cross-derivatives (denoted by $M_\times$) as

$$\text{colsp}(\mathbf{Q}) = \text{colsp}\begin{pmatrix}\mathbf{M}_s & \mathbf{M}_{\lambda_1} & \ldots & \mathbf{M}_{\lambda_n} & \mathbf{M}_\times\end{pmatrix}. \tag{3}$$

In general it is not essential to include cross-derivatives inorder to preserve the accuracy of typical electrical/thermal subcircuits. It is to be noted that once computed, the multidimensional subspace $\mathbf{Q}$ is a constant real matrix independent of frequency and $\lambda_1 \ldots \lambda_n$.

C. Macromodeling through Congruent Transformation

The multidimensional subspace formed in the previous subsection is used to perform a congruent transformation on the original system to produce a reduced-order macromodel. Define

$$\hat{x} = \mathbf{Q}x \tag{4}$$

where $\hat{x} \in \Re^q$ and q is the reduced vector of unknowns. It has to be noted that $q \ll N$.

Using (4) a congruent transformation is performed on (1) to give

$$\hat{\mathbf{C}}(\lambda_1 \ldots \lambda_n)\frac{d\hat{x}}{dt} + \hat{\mathbf{G}}(\lambda_1 \ldots \lambda_n)\hat{x} = \hat{\mathbf{B}}u_p(t) \tag{5a}$$

$$i_p(t) = \hat{\mathbf{B}}^T\hat{x} \tag{5b}$$

where,

$$\hat{\mathbf{C}} = \mathbf{Q}^T\mathbf{C}\mathbf{Q}; \quad \hat{\mathbf{G}} = \mathbf{Q}^T\mathbf{G}\mathbf{Q}; \quad \hat{\mathbf{B}} = \mathbf{Q}^T\mathbf{B} \tag{6}$$

It is to be noted that the size of the macromodel (5) is very small compared to the original network (1). The response at any node of the original network can be computed using (4) once the solution of the reduced system (5) is known.

It can be proved that the time-domain macromodel so formed preserves the dominant eigen-values of the original network (1). In addition if the original network is passive, it can also be proved that the macromodel (5) is also passive. However, these proofs have been omitted due to lack of space.

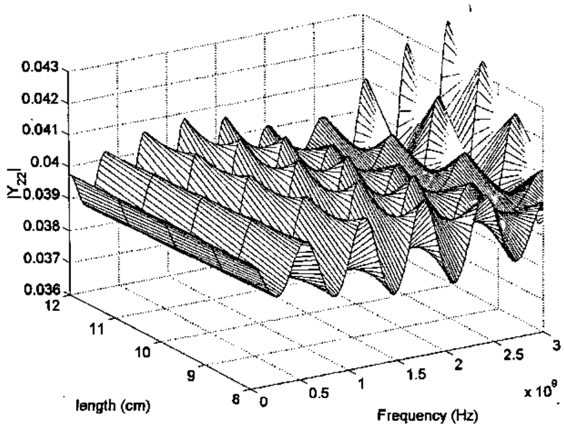
III. Results

A. Example 1: A Transmission Line Macromodel

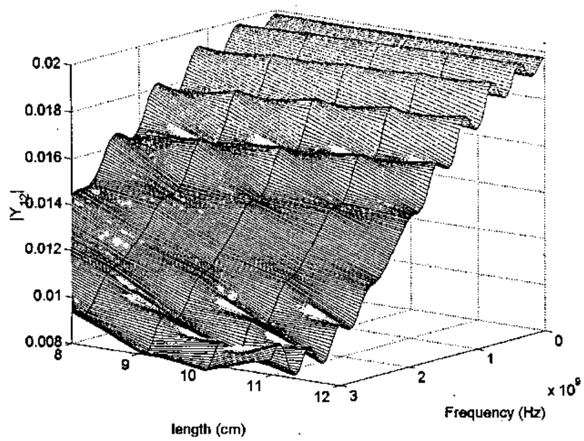
A distributed transmission line network with two input/output ports containing a set of nine coupled transmission lines [12] and RLC components was chosen to demonstrate the efficiency and accuracy of the proposed algorithm. The distributed network was discretized using lumped segmentation. The size of the MNA (Modified Nodal Analysis) equations representing this network was 3612×3612. The proposed algorithm can be used to form a macromodel of the distributed network as a function of time/frequency and any design parameter of the network. For this specific example, the length of the transmission line was taken as the design parameter of interest. Block moments w.r.t frequency as well as the length of the transmission lines were considered to form the reduced-order macromodel. The size of the equations representing the reduced macromodel was 150×150. The CPU cost of performing the reduction was 21.43 s. Note that this is a one time cost to form the reduced model and does not need to be repeated for each new frequency point or line length. A comparison of the Y-parameters of the original network and the reduced macromodel are shown in Figs. 1a and 1b. The Y-parameters obtained from the reduced macromodel match the results of the original system accurately (error within 0.26%). For this analysis, 512 frequency points were taken at 9 different values of the line length ranging from 8 to 12cm. A comparison of the CPU time taken to generate the responses in Fig. 1 is shown in Table I. A speed up of 11.48 was achieved using the proposed algorithm.

B. Example 2: A Thermal Macromodel of a GaAs Power Amplifier

In the second example, the theory developed in this paper is applied to a thermal model. The thermal model was built



a) $|Y_{22}|$



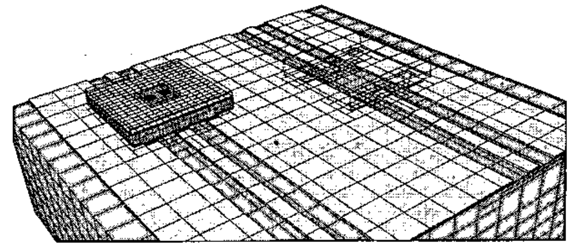
b) $|Y_{12}|$

Fig. 1. $|Y_{22}|$ and $|Y_{12}|$ of the interconnect macromodel are shown as the length of the transmission line is varied from 8cm to 12cm.

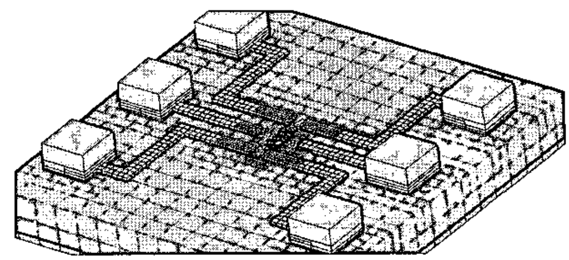
	Size	Sim. Time	Speed-up
Org. System	3612×3612	2916 s	–
Red. System	150×150	254 s	11.48

TABLE I

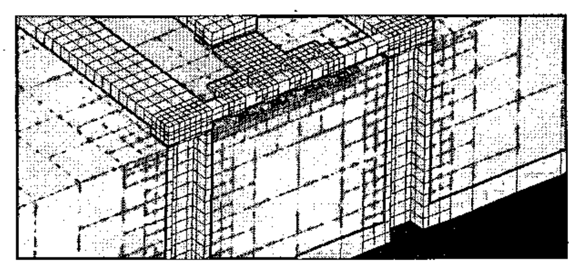
CPU COMPARISON OF THE TRANSMISSION LINE EXAMPLE



a) Substrate and two amplifiers



b) Amplifier



c) Cross-sectional view of amplifier

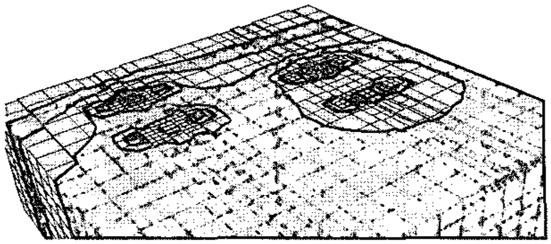
Fig. 2. Atar models showing physical structure and mesh

using *Atar* [13], a 3D thermal multiscale modeling tool based on finite differences using a quad tree mesh. *Atar* builds the model directly from layout and technology information. As a demonstration, a model of two GaAs amplifiers attached to a ceramic substrate, was built. Fig. 2a shows a view of the *Atar* model of the substrate and the two amplifiers (one of the amplifiers is only shown in outline). In order to correctly determine the internal temperatures of the GaAs power amplifier it needs to be modeled in considerable detail (see Figs. 2b and 2c) requiring around 40,000 blocks for each amplifier. The amplifier consists of four 32 μm long transistors formed in a mesa structure on a 1mm square GaAs chip 100 μm thick, included in the model are six 60 μm gold bumps for attachment to a substrate, and all emitter, base and collector metalization. The metalization includes local heat spreaders and thru-vias to the backside.

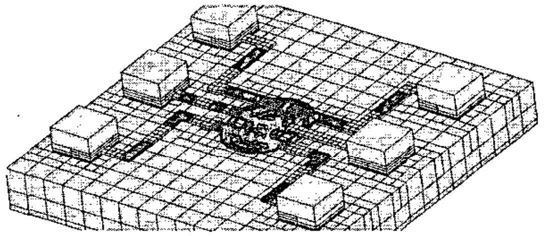
To facilitate quick simulation of the microwave circuit, a thermal macromodel was created for the GaAs power amplifier. In the macromodel the 6 interconnect bumps were designated as thermal ports and the design parameters were the transistor power levels and the heat flow from the backside of the chip (which of course would be determined by the packaging configuration). With these considerations Atar produces the matrices $\mathbf{G}$ and $\mathbf{C}$ which represent the discretized thermal resistance and capacity respectively. Two parameters were used to capture the heat flow off the backside of the wafer. The first $\lambda_\alpha(W/\mu m^2K)$ represents the thermal resistance to the ambient and the second, the ambient temperature $T_{BC}(K)$.

When the proposed algorithm was applied to the *Atar* GaAs amplifier, a macromodel of size 54 with 6 ports was created. The model was tested by running steady state simulations and varying λ_α from zero (no heat flow) to a large value of 10^{-3} $W/\mu m^2K$ (a fixed temperature BC). In Fig. 3 three views of the thermal distribution are shown for a large value of λ_α. The amplifier temperatures obtained from the reduced-order macromodel were found to be within 2% of the value obtained by full simulation.

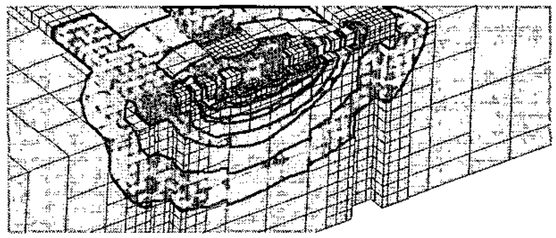
The full solution of the substrate and amplifiers (90,000 blocks) took 600 s. This can be contrasted to the solution of the substrate (4000 blocks) and two macromodeled amplifiers which took 0.18 s. The model reduction of the amplifier took 300 s. A primary advantage of the multidimensional macromodeling technique is that new simulations for different values of λ_α do not require a new model reduction step. During a design cycle this can have a significant effect on efficiency. As an example, if a designer was to test 10 values of λ_α and 10 different ambient temperatures we can calculate the CPU cost for both approaches. For full simulation, the CPU cost would be $600 \times 100 = 60,000$ s. Whereas the use of macromodels would require $300 + 100 \times 0.18 = 318$ s; for a resultant speed-up of 188.



a) Thermal contours in substrate



b) Thermal contours across amplifier



c) Thermal contours in cross-section

Fig. 3. Atar models showing the thermal distribution.

IV. CONCLUSIONS

This paper presents an new technique that forms reduced-order macromodels which model the original network w.r.t time as well as multiple design parameters. The proposed algorithm was tested on several large networks to form multidimensional macromodels. The size of the reduced models were less than 5% when compared to the original circuit.

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